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NRQCD matrix elements in polarization of J/Psi produced from b-decay

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We present the non-relativistic QCD (NRQCD) prediction for the polarization of the J/ψ produced in $b \to J/\psi + X$, as well as the helicity-summed production rate. We propose that these observables provide a means of measuring the three most important color-octet NRQCD matrix elements involved in J/ψ production. Anticipating the measurement of the polarization parameter α, we determine its expected range given current experimental bounds on the color-octet matrix elements.

I. INTRODUCTION

A rigorous theoretical framework within which quarkonium production can be studied is provided by the nonrelativistic QCD (NRQCD) factorization formalism developed by Bodwin, Braaten, and Lepage [1]. This approach is based on NRQCD [2], an effective field theory that can be made equivalent to full QCD to any desired order in heavy-quark relative velocity. The NRQCD factorization formalism is not a model, but rather a rigorous derivation within NRQCD of a factorized form for quarkonium production and decay rates.

A central result of the NRQCD factorization formalism is that inclusive quarkonium production cross sections must have the form of a sum of products of short-distance coefficients and NRQCD matrix elements. The short-distance coefficients are associated with the production of a heavy quark-antiquark pair in a specific color and angular-momentum state. They can be calculated using ordinary perturbative techniques. As to the NRQCD matrix elements,

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they parameterize the effect of long-distance physics such as the hadronization of the quark-antiquark pair. These can be determined phenomenologically.

The power of the NRQCD formalism stems from the fact that factorization formulas for observables are essentially expansions in the small parameter v^2, where v is the average relative velocity of the heavy quark and anti-quark in the quarkonium bound state. $v^2 \sim 0.3$ for charmonium, and 0.1 for bottomonium. NRQCD v-scaling rules [3] allow us to estimate the relative sizes of various NRQCD matrix elements. This information, along with the dependence of the short-distance coefficients on coupling constants, permits us to decide which terms must be retained in expressions for observables so as to reach a given level of accuracy. Generally, to leading order, factorization formulas involve only a few matrix elements, so several observables can be related by a small number of parameters.

NRQCD predictions therefore presuppose QCD, factorization, and a reasonably convergent v^2 expansion. If the quantitative predictions of NRQCD fail, one of these three assumptions is failing, most likely the validity of the v^2 expansion.

In many instances of J/ψ production, the most important NRQCD matrix elements are $\langle \mathcal{O}_1^\psi(^3S_1)\rangle$, $\langle \mathcal{O}_8^\psi(^3S_1)\rangle$, $\langle \mathcal{O}_8^\psi(^1S_0)\rangle$, and $\langle \mathcal{O}_8^\psi(^3P_J)\rangle$. These four quantities parametrize the hadronization into a J/ψ boundstate of a $c\bar{c}$ pair produced initially with the stated quantum numbers (angular momentum $^{2S+1}L_J$, color quantum number 1 or 8). Previous to the development of the factorization formalism of Ref. [1], most J/ψ production calculations took into account only the hadronization of $c\bar{c}$ pairs initially produced in the color-singlet $^3S_1$ state, as parameterized by $\langle \mathcal{O}_1^\psi(^3S_1)\rangle$.

Recently, in regard to the three most important color-octet matrix elements involved in J/ψ production, it has been noticed that there prevails a certain inconsistency between the values measured at CDF [4] and those measured in other processes [5]. We propose here that inclusive J/ψ production from b-decay can be of avail in unravelling this difficulty by serving as a supplementary arena for measuring the contentious matrix elements.

Inclusive production of J/ψ from b-decay provides two measurable combinations of the matrix elements. The first one is the helicity-summed rate $\Gamma(b \to J/\psi + X)$. The second combination concerns the polarization parameter α appearing in the electromagnetic decay rate of J/ψ to lepton pairs:

$$\frac{d\Gamma}{dcos\theta}\big(\psi \to \mu^+\mu^-(\theta)\big) \propto 1 + \alpha\cos^2\theta \,, \tag{1}$$

where the polar angle θ is defined in the J/ψ rest frame for which the z-axis is aligned with the direction of motion of the J/ψ in the lab.

The short-distance physics for the process $b \to J/\psi + X$ is described by the four-quark Fermi interactions $b \to c\bar{c}s$ and $b \to c\bar{c}d$, at the m_b scale. The Feynman diagram is shown in Fig. 1. Let us define P to be the total four-momentum of the $c\bar{c}$ system. The s and d quarks are assumed massless.

II. FACTORIZATION FORMULA FOR THE SPIN-SUMMED PRODUCTION RATE

We first calculate the factorization formula for the spin-summed J/ψ production rate. Let us define $\mathbf{q}$ as the relative momentum of the c and $\bar{c}$ quarks evaluated in the rest frame of the $c\bar{c}$ pair. The NRQCD matching formalism requires only that we know the algebraic form of the amplitude for small values of $\mathbf{q}$. This is because the hadronization of the heavy quark pair into quarkonium occurs with nonnegligible probability only when $|\mathbf{q}| \ll m_c$. Moreover, this is the regime in which the NRQCD effective lagrangian is valid. Therefore, we expand the Feynman amplitude in powers of $\mathbf{q}$ and keep only pieces up to linear order. The amplitude for $b \to c\bar{c}s$ is given by

$$\begin{aligned}\mathcal{M}(\sigma,\tau;c,d,e,f;\mathbf{q}) \;&=\; \frac{G_F}{\sqrt{2}}\, V_{cb}V_{cs}^* \left[\frac{(2C_+ - C_-)}{3}\delta^{cd}\delta^{ef} \;+\; (C_+ + C_-)T^g_{cd}T^g_{ef}\right]\; \bar{u}_s\gamma^\mu(1-\gamma_5)u_b \\ &\times \left[2m_c\;\Lambda^\mu_i \xi^\dagger_\sigma\sigma^i\eta_\tau \;-\; P^\mu\;\xi^\dagger_\sigma\eta_\tau \;+\; 2i\Lambda^\mu_k\epsilon^{mik}\;\xi^\dagger_\sigma q^m\sigma^i\eta_\tau\right]\left(1 + O\left(\frac{\mathbf{q}^2}{m_c^2}\right)\right)\,,\end{aligned} \tag{2}$$

where ξ and η are non-relativistic two-spinors for the c and $\bar{c}$ quarks respectively; c,d,e,f are color indices and σ and τ are the individual spins of the charm quarks. The factors involving C_+ and C_- are Wilson coefficients which govern the scale-evolution of the four-quark Fermi interaction. Λ^μ_i (defined in detail in Ref. [6]) is the Lorentz boost matrix that takes a three-vector from the $c\bar{c}$ rest frame to the lab frame . In the above equation, the term with the color Krönecker deltas contributes to the production of $c\bar{c}$ pairs in color-singlet states, and the term with color matrices T^g contributes to the production of $c\bar{c}$ pairs in color-octet states.

From Eq. 2, using the matching procedure presented in Ref [7], we determine the spin-summed rate for $b \to J/\psi + X$ in the NRQCD factorization formalism:

$$\Gamma(b \to J/\psi + X) = \frac{G_F^2}{864\pi} \frac{(m_b^2 - 4m_c^2)^2}{m_b^3 m_c} \Big|V_{cb}\Big|^2 \Bigg(2(2C_+ - C_-)^2 (m_b^2 + 8m_c^2) \big\langle \mathcal{O}_1^\psi(^3S_1) \big\rangle$$
$$+ \quad 3(C_+ + C_-)^2 (m_b^2 + 8m_c^2) \big\langle \mathcal{O}_8^\psi(^3S_1) \big\rangle \quad + \quad 9(C_+ + C_-)^2 m_b^2 \big\langle \mathcal{O}_8^\psi(^1S_0) \big\rangle$$
$$+ \quad 6(C_+ + C_-)^2 (m_b^2 + 8m_c^2) \frac{\big\langle \mathcal{O}_8^\psi(^3P_1) \big\rangle}{m_c^2} \Bigg) \Big(1 + O(v^2)\Big) \tag{3}$$

where, upon summing over the light quarks s and d, we have used $|V_{cb}V_{cs}^*|^2 + |V_{cb}V_{cd}^*|^2 \approx |V_{cb}|^2$. We have taken the inital b-quark to be unpolarized. Here $\langle \mathcal{O}_n^\psi \rangle \equiv \langle 0 | \mathcal{O}_n^\psi | 0 \rangle$ are NRQCD J/ψ production matrix elements. We consider only the leading color-singlet piece and the leading color-octet pieces in the relativistic v^2-expansion. Our calculation of the coefficients in the above spin-summed factorization formula concurs with the results presented in Ref. [8].

According to the NRQCD v-scaling rules, the color-octet matrix elements in Eq. 3 are all expected to be suppressed by v^4 with respect to the basic color-singlet matrix element $\langle \mathcal{O}_1^\psi(^3S_1) \rangle$. Thus, at the outset, one would not expect the octet contributions to play a major role in the production of J/ψ articles. However, as pointed out in Ref. [9], the color-singlet coefficient $2C_+ - C_-$ decreases significantly as it is evolved down from its value of 1 at the scale M_W to its value of roughly 0.40 at the scale m_b. On the other hand, the color-octet coefficient $(C_+ + C_-)/2$ increases slightly from 1 at the scale M_W to roughly 1.10 at the scale m_b. For this reason, the octet contributions are actually as important as, or more important than, the basic singlet contribution. In fact, the short-distance coefficients in the color-octet terms are some 50 times larger than those in the singlet term!

III. FACTORIZATION FORMULA FOR HELICITY RATES

In the previous section, we discussed how the serendipitous Wilson-coefficient enhancement of the octet contributions enables us to measure the color-octet matrix elements contributing to the helicity-summed rate $\Gamma(b \to J/\psi + X)$. This enhancement can be further exploited to determine the color-octet matrix elements by considering the polarization of the produced J/ψ's, as measured by the parameter α in Eq. 1 . It can be calculated via the formula

$$\alpha = \frac{\sigma(+) + \sigma(-) - 2\sigma(0)}{\sigma(+) + \sigma(-) + 2\sigma(0)} \tag{4}$$

where $\sigma(\lambda)$ represents the production rate of J/ψ with helicity λ.

Since α is a ratio, we need only calculate the relative sizes of the various contributions to the helicity rates. Thus, we need only calculate the effective Feynman amplitude squared, evaluated in the b rest frame, with the J/ψ momentum $\mathbf{P}$ oriented in the positive z-direction. In this case, the angular momentum of the J/ψ in the canonical z-direction corresponds simply to the particle's helicity.

Beneke and Rothstein [10] and Braaten and Chen [6] have developed techniques for deriving the production rates of quarkonia with specified helicities. Applying their methods to the amplitude given in Eq. 2, we obtain

$$\Gamma\big(b \to J/\psi(\lambda) + X\big) \propto 2(2C_+ - C_-)^2 \big\langle \mathcal{O}_1^\psi(^3S_1) \big\rangle \left[(m_b^2 - 4m_c^2)\delta_{\lambda 0} + 4m_c^2(1-\lambda)\right]$$
$$+ 3(C_+ + C_-)^2 \big\langle \mathcal{O}_8^\psi(^3S_1) \big\rangle \left[(m_b^2 - 4m_c^2)\delta_{\lambda 0} + 4m^2(1-\lambda)\right]$$
$$+ 3(C_+ + C_-)^2 \big\langle \mathcal{O}_8^\psi(^1S_0) \big\rangle m_b^2$$
$$+ 9(C_+ + C_-)^2 \frac{\big\langle \mathcal{O}_8^\psi(^3P_0) \big\rangle}{m_c^2} \left[(m_b^2 - 4m_c^2)(1 - \delta_{\lambda 0}) + 8m_c^2 - 4m_c^2\lambda\right] . \tag{5}$$

In displaying Eq. 5, we have adopted the standard practice of rewriting the matrix element $\langle \mathcal{O}_8(^3P_1) \rangle$ using the relation

$$\big\langle \mathcal{O}_8^\psi(^3P_J) \big\rangle \simeq (2J+1) \big\langle \mathcal{O}_8^\psi(^3P_0) \big\rangle. \tag{6}$$

This latter relation is due to heavy-quark spin symmetry, and is valid up to relative order v^2. It must be kept in mind, however, that b-decay does not actually produce $c\bar{c}$ states in the $^3P_0$ configuration at leading order in the coupling constants.

Some consequences of angular momentum conservation and of the left-handedness of the charged-current Fermi interaction are illustrated in Fig. 2. The thin arrow labelled $\overrightarrow{\mathbf{P}}$ represents the trajectory of the center of mass of

the $c\bar{c}$ system. The other thin arrow represents the trajectory of the s or d quark. Thick arrows denote intrinsic angular momenta, with $\lambda_{c\bar{c}}$ denoting the helicity of the $c\bar{c}$ system prior to hadronization. Since the charged-current Fermi interaction couples only the left-handed parts of the fermion fields, the masslessness of the s and d quarks ensures that they are emitted with negative helicity. Therefore, due to angular momentum conservation, the helicity outcome $\lambda_{c\bar{c}} = +1$ is not allowed. This has certain obvious consequences for the J/ψ helicity production rates: the basic color-singlet contribution (parametrized by $\langle \mathcal{O}_1^{\psi}(^3S_1)\rangle$ and involving the direct hadronization of a color-singlet $^3S_1$ state into a J/ψ) is zero for $\lambda = +1$, at lowest order in the relativistic expansion. The same is also true of the $\langle \mathcal{O}_8^{\psi}(^3S_1)\rangle$ contribution, which involves the hadronization of a color-octet $^3S_1$ state into a J/ψ via the heavy-quark-spin-preserving $L = 0$ emission or absorption of two soft gluons [11].

The color-singlet contribution to the helicity rates of the J/ψ produced in b-decay was calculated in the color-singlet model by M. Wise in Ref. [12] 16 years ago. This author presented expressions for the production rates of longitudinal and transverse helicities, given in terms of the color-singlet radial wave function at the origin. The result in Ref. [12] concurs with the first line of our Eq. 5.

IV. PREDICTION FOR THE POLARIZATION PARAMETER ALPHA

While experimental determinations of helicity-summed $\Gamma(b \to J/\psi + X)$ have already been carried out [13], a measurement of the polarization parameter α is not yet available. Anticipating the availability of this latter measurement, it is interesting to determine the range of α which is predicted by the NRQCD factorization formalism. Using Eqs. 4 and 5, we can express α in terms of the NRQCD matrix elements. Before proceding, however, we must first decide which value of m_b to use. Since we are presenting a leading-order calculation, we may in principle choose to use either the pole mass or the $\overline{\mathrm{MS}}$ mass for m_b, the difference between these two being merely a higher-order-in-α_s effect. However, it turns out that the leading-order NRQCD prediction of the polarization parameter α depends quite strongly on the choice of m_b. Therefore we report results for a wide range of m_b, from 4.1 GeV to 5.3 GeV. This range includes the values of $m_b = 4.3 \pm 0.2$ GeV corresponding to the $\overline{\mathrm{MS}}$ mass, and the values $m_b = 5.0 \pm 0.2$ GeV corresponding to the pole mass determined on the lattice [14]. Our range is centered around $m_b = 4.7$ GeV, which is the central value for the pole mass [13]. This choice is consistent with the fact that the numerical value that we use for $\langle \mathcal{O}_1^{\psi}(^3S_1)\rangle$ is taken from Ref. [14], in which the pole mass was used in the extraction of the matrix element value.

For concreteness, we present in the text below only our results for the choice $m_b = 4.7$ GeV. Using $C_+(m_b) = 0.868$, $C_-(m_b) = 1.329$ and $m_c = 1.55$ GeV, one obtains the leading-order NRQCD prediction for the polarization parameter

$$\alpha = \frac{-0.39\big\langle \mathcal{O}_1^{\psi}(^3S_1)\big\rangle - 17\big\langle \mathcal{O}_8^{\psi}(^3S_1)\big\rangle + 52\big\langle \mathcal{O}_8^{\psi}(^3P_0)\big\rangle/m_c^2}{\big\langle \mathcal{O}_1^{\psi}(^3S_1)\big\rangle + 44\big\langle \mathcal{O}_8^{\psi}(^3S_1)\big\rangle + 61\big\langle \mathcal{O}_8^{\psi}(^1S_0)\big\rangle + 211\big\langle \mathcal{O}_8^{\psi}(^3P_0)\big\rangle/m_c^2} \,. \tag{7}$$

It must be noted that α depends only weakly on $\langle \mathcal{O}_8^{\psi}(^3S_1)\rangle$ and most strongly on $\langle \mathcal{O}_8^{\psi}(^3P_0)\rangle/m_c^2$.

We now procede to determine the range of α which is consistent with existing information on the matrix elements $\langle \mathcal{O}_1^{\psi}(^3S_1)\rangle$, $\langle \mathcal{O}_8^{\psi}(^3S_1)\rangle$, $\langle \mathcal{O}_8^{\psi}(^1S_0)\rangle$, and $\langle \mathcal{O}_8^{\psi}(^3P_0)\rangle/m_c^2$ and with various constraints on linear combinations of these quantities. We first review this information:

- Bodwin et al. [14] have determined the color-singlet matrix element $\langle \mathcal{O}_1^{\psi}(^3S_1)\rangle$ from decay rates of J/ψ to lepton pairs to be $\langle \mathcal{O}_1^{\psi}(^3S_1)\rangle = 1.1 \pm 0.1$ GeV^3. The error reflects theoretical uncertainties due to higher order α_s and v^2 corrections. This phenomenologically extracted value for the color-singlet matrix element is in agreement with the lattice calculation result also presented in Ref. [14].

- A constraint on the octet matrix elements is given by the requirement that the theoretical prediction for the spin-summed production rate $\Gamma(b \to J/\psi + X)$ given in Eq. 3 be consistent with the experimentally measured value of 3.42 μeV. This constraint can be expressed as

$$0.342 \text{ GeV}^3 = 0.096\big\langle \mathcal{O}_1^{\psi}(^3S_1)\big\rangle + 4.21\big\langle \mathcal{O}_8^{\psi}(^3S_1)\big\rangle + 6.76\big\langle \mathcal{O}_8^{\psi}(^1S_0)\big\rangle + 25.3\frac{\big\langle \mathcal{O}_8^{\psi}(^3P_0)\big\rangle}{m_c^2} \,. \tag{8}$$

  To compute the above experimental value of $\Gamma(b \to J/\psi + X) = 3.42\mu$eV, we have used $BR\big(B(\text{charge not determined}) \to J/\psi(\text{direct}) + X\big)$ =(0.80±0.08)%, and τ(averaged over B hadrons)=1.54 picoseconds. To compute the short-distance coefficients on the right-hand-side of Eq. 8, we have taken $G_F = 1.1664 \times 10^{-5}$ GeV^{-2} [13] and $|V_{cb}| = 0.0381 \pm 0.0021$ [15]. One can attribute an error of roughly 30% to the above equation, to reflect relativistic corrections and uncertainties in the scale used to evaluate the

Wilson coefficients [9,16]. It must be pointed out that the color-singlet model prediction for spin-summed J/ψ production — obtained by setting all color-octet matrix elements to zero, and using $\langle\mathcal{O}_1^{\psi}({}^3S_1)\rangle = 1.1 \pm 0.1$ GeV^3 — is roughly one-third of the experimental value. However the NRQCD prediction for the b-decay width can be made to agree with the experimental measurements for reasonable values of the color-octet matrix elements.

- Cho and Leibovich [4] have performed a fit to CDF data over low and high p_T ranges and have found

$$\big\langle\mathcal{O}_8^{\psi}({}^3S_1)\big\rangle = 0.0066 \pm 0.0021 \text{ GeV}^3 \tag{9}$$

and

$$\big\langle\mathcal{O}_8^{\psi}({}^1S_0)\big\rangle + 3\frac{\big\langle\mathcal{O}_8^{\psi}({}^3P_0)\big\rangle}{m_c^2} = 0.066 \pm 0.015 \text{ GeV}^3 \ . \tag{10}$$

- From photoproduction data, Amundson et al. [17] have determined

$$\big\langle\mathcal{O}_8^{\psi}({}^1S_0)\big\rangle + 7\frac{\big\langle\mathcal{O}_8^{\psi}({}^3P_0)\big\rangle}{m_c^2} = 0.020 \pm 0.001 \text{ GeV}^3 \ . \tag{11}$$

  Note that the errors quoted in Eqs. 9, 10, and 11 are statistical only, and that the analyses in [4] and [17] did not take into account theoretical uncertainties due to next-to-leading order (NLO) corrections. However we expect NLO corrections to be very large (see [18] for a discussion of NLO corrections in hadronic J/ψ production calculations). Moreover the photoproduction results of Ref. [17] may also have significant higher-twist corrections [10].

- Finally, due to v-scaling arguments, we expect the color-octet matrix elements to be roughly of order $v^4\langle\mathcal{O}_1^{\psi}({}^3S_1)\rangle$.

With this information in mind, we now determine the range of values of α that we can expect. Our method of determining this range consists of "scanning" through the three-dimensional parameter space of the color-octet matrix elements, and determining the maximum and minimum value of α that occur in the allowed volume. Our scan is subject to the following constraints: we allow $\langle\mathcal{O}_8^{\psi}({}^3S_1)\rangle$ to vary in the range

$$\big\langle\mathcal{O}_8^{\psi}({}^3S_1)\big\rangle \in [0.003, .014] \text{ GeV}^3 \ ; \tag{12}$$

we allow the photoproduction-related linear combination of $\langle\mathcal{O}_8^{\psi}({}^1S_0)\rangle$ and $\langle\mathcal{O}_8^{\psi}({}^3P_0)\rangle$, to vary in the range

$$\big\langle\mathcal{O}_8^{\psi}({}^1S_0)\big\rangle + 7\frac{\big\langle\mathcal{O}_8^{\psi}({}^3P_0)\big\rangle}{m_c^2} \in [0.0, 0.04] \text{ GeV}^3 \ , \tag{13}$$

where the range we have chosen takes into account theoretical uncertainties due to NLO corrections and possibly significant hight-twist corrections; we allow the b-decay-related linear combination given in Eq. 8 to vary $\pm 30\%$ about the central value of $0.342\,\text{GeV}^3$, *i.e.*

$$0.096\big\langle\mathcal{O}_1^{\psi}({}^3S_1)\big\rangle + 4.21\big\langle\mathcal{O}_8^{\psi}({}^3S_1)\big\rangle + 6.76\big\langle\mathcal{O}_8^{\psi}({}^1S_0)\big\rangle + 25.3\frac{\big\langle\mathcal{O}_8^{\psi}({}^3P_0)\big\rangle}{m_c^2} \in [0.24, 0.45] \text{ GeV}^3 \ , \tag{14}$$

where the range we have chosen takes into account theoretical uncertainties due to v^2 relativistic corrections as mentioned previously. Note that the above constraints are sufficient to insure that the helicity rates in Eq. 5 are positive for each value of the helicity λ.

To the above experimental constraints on the allowed parameter space, we must of course add the theoretical requirement that the absolute values of the octet matrix elements respect v-scaling rules, *i.e.* that they must not be unreasonably larger than $v^4\langle\mathcal{O}_1^{\psi}({}^3S_1)\rangle \sim 0.1$. Moreover, we must impose on our allowed volume of parameter space that the matrix element $\langle\mathcal{O}_8^{\psi}({}^1S_0)\rangle$ always be positive. No similar constraint need to be applied to the matrix element $\langle\mathcal{O}_8^{\psi}({}^3P_0)\rangle$. Indeed, it can be negative, for the following reason. The bare matrix element (which is of course positive definite) contains a power ultraviolet divergence. Such a power ultraviolet divergence must be proportional to a matrix element of an operator of lower dimension. In the case of $\langle\mathcal{O}_8^{\psi}({}^3P_0)\rangle/m_c^2$, the divergence is proportional

to $\langle\mathcal{O}_1^{\psi}(^3S_1)\rangle$ [1]. This divergence must be subtracted to obtain the renormalized matrix element. Since the piece subtracted is comparable in magnitude to the bare matrix element, the difference can be negative. On the other hand, since the operator $\mathcal{O}_8^{\psi}(^1S_0)$ belongs to the set of lowest dimension NRQCD four-fermion operators, the bare matrix element $\langle\mathcal{O}_8^{\psi}(^1S_0)\rangle$ cannot have power divergences. Thus the subtractions involved in renormalization cannot transform the positive definite bare matrix element into a negative quantity [19]. Due to these considerations, we therefore impose the additional theoretical constraint

$$\langle\mathcal{O}_8^{\psi}(^1S_0)\rangle > 0. \tag{15}$$

The constraints expressed in eqs. 12, 13, 14 and 15 together define the volume in the three-dimensional parameter space that is allowed by current experimental information and theoretical considerations.

In defining the allowed parameter space, we have not used an experimental constraint on the CDF-related linear combination presented in Eq. 10. Interestingly, we find that with the imposition of all the above constraints (Eqs. 12 through 15), this combination is limited to take values in the range

$$\left\langle\mathcal{O}_8^{\psi}(^1S_0)\right\rangle + 3\frac{\left\langle\mathcal{O}_8^{\psi}(^3P_0)\right\rangle}{m_c^2} \in [0.01, 0.06]\ \mathrm{GeV}^3\ . \tag{16}$$

The central value of 0.066 quoted in Eq. 10 falls just outside the upper end of this range, indicating a possible inconsistency between the result of Ref. [4] and the results from other processes considered in the present analysis.

We now present our main result, which is the expected range of α: The *maximum* value for α (-0.09) is obtained when $\langle\mathcal{O}_8^{\psi}(^3S_1)\rangle$ is at the minimum of its allowed range and when the photoproduction-related combination $\langle\mathcal{O}_8^{\psi}(^1S_0)\rangle + 7\langle\mathcal{O}_8^{\psi}(^3P_0)\rangle/m_c^2$ is at the maximum of its allowed range. The *minimum* value for α (-0.33) occurs in the opposite situation.

So far in our discussion, we have considered only the particular choice $m_b = 4.7$ GeV. Table I shows how our findings depend on m_b. The main results are the NRQCD predictions for the range of α. Alongside the full NRQCD results, we display the color-singlet model predictions for α. The error bars on the color-singlet-model predictions reflect v^4 relativistic corrections; the relativistic corrections to the color-singlet result are of order v^4, not v^2 as one might expect *a priori*, because the v^2 corrections to the helicity rates factor and cancel in the ratio α. In the other three columns of Table I, we treat the quantities appearing at the top of the columns as functions of the variables through which we scan; we report the ranges of values of these quantities that occur for the allowed parameter space.

Our leading-order results depend strongly on the values taken for m_b and m_c. As stated previously we have used $m_c = 1.55$ GeV to generate the results given in Table I. For some optional choices of m_c we obtain the following ranges of α. Choosing $m_b = 4.7$ GeV and $m_c = 1.25$ GeV we find that $\alpha \in [-.53, -.12]$. Choosing $m_b = 4.7$ GeV and $m_c = 1.85$ GeV we find that $\alpha \in [-.22, -.13]$.

In general for higher values of m_b the predicted range for α enlarges. For increasing values of m_c the minimum value we predict for α becomes greater, while the maximum value remains roughly unchanged.

A general conclusion that can be drawn by looking at Table I is that the inclusion of octet matrix elements *raises* the predicted range for α significantly from the color-singlet prediction.

V. CONCLUSION

We have proposed that a measurement of the polarization of J/ψ particles produced in the process $b \to J/\psi + X$ can furnish a useful new means of determining the color-octet matrix elements involved in J/ψ production. This measurement, which is not yet available, will supplement existing experimental information, which includes extractions from CDF, photoproduction, and the helicity-summed rate $\Gamma(b \to J/\psi + X)$. Further experimental information would be of great utility, since there currently prevails a certain inconsistency between the values of the matrix elements measured at CDF and those values extracted from other processes.

We have presented the factorization formula for the helicity-summed rate in Eq. 3. Rates for specified J/ψ helicity have been calculated using the methods of Beneke and Rothstein [10] and Braaten and Chen [6], and are presented in Eq. 5. The resulting expression for the polarization parameter α is given in Eq. 7 (for the choice $m_b = 4.7$ GeV).

In anticipation of the measurement of the polarization parameter α, we have found it interesting to determine the range of α which is predicted by the NRQCD factorization formalism, given current experimental information on the NRQCD matrix elements. We find that, for $m_b = 4.7$ GeV, α is expected to range from -0.33 to -0.09. Our method of determining this range consists of "scanning" through the three-dimensional parameter space of the color-octet matrix elements that is allowed by the constraints expressed in Eqs. 12, 13, 14 and 15. Upon inspecting the

m_b (GeV)	color-singlet model predictions for α	NRQCD factorization formalism predictions for α	range of $\langle\mathcal{O}_8^\psi(^1S_0)\rangle + 3\frac{\langle\mathcal{O}_8^\psi(^3P_0)\rangle}{m_c^2}$ (GeV^3)	range of $\langle\mathcal{O}_8^\psi(^1S_0)\rangle$ (GeV^3)	range of $\frac{\langle\mathcal{O}_8^\psi(^3P_0)\rangle}{m_c^2}$ (GeV^3)
4.1	$-.27 \pm .03$	$[-.23,\ -.14]$	[.08,.2]	[.1,.4]	[−.06,−.009]
4.4	$-.34 \pm .03$	$[-.28,\ -.11]$	[.03,.1]	[.02,.2]	[−.03,.003]
4.7	$-.40 \pm .04$	$[-.35,\ -.08]$	[.009,.06]	[0,.1]	[−.01,.006]
5.0	$-.45 \pm .05$	$[-.42,\ -.09]$	[.002,.03]	[0,.06]	[−.008,.006]
5.3	$-.49 \pm .05$	$[-.47,\ -.12]$	[.001,.02]	[0,.03]	[-.005,.005]

TABLE I. Theoretical leading-order predictions for the expected range of α. We present predictions for α in the color-singlet model and in the NRQCD factorization formalism. The ranges are determined by "scanning" through the allowed volume in the three-dimensional parameter space constrained by Eqs. (12), (13), (14) and (15) and by finding within that volume the maximum and minimum occuring values of α. Results are given for various values of m_b. Also given, in the last three columns, are the ranges of the CDF-related combination and of the two matrix elements indicated, resulting from the set of constraints.

parameter-space volume so defined, we find that the linear combination of matrix elements that has been measured at CDF [4], namely $\langle\mathcal{O}_8^\psi(^1S_0)\rangle + 3\langle\mathcal{O}_8^\psi(^3P_0)\rangle/m_c^2$ is limited to take values from 0.01 to 0.06 GeV^3, a situation which is at the threshold of incompatibility with the value of 0.066 ± 0.015 reported in Ref. [4].

In Table I, we have shown how our leading order predictions depend on m_b. In general, it can be concluded that the inclusion of octet matrix elements raises the predicted range for α significantly from the color-singlet prediction, and that the range of α broadens as m_b is increased.

Unfortunately, due to the large number of poorly determined parameters, including m_b and m_c, the expected range of α as predicted by NRQCD is large.

It may be that an accurate NRQCD prediction of the polarization parameter (beyond leading order) will not be possible for a long time. This possibility, however, does not degrade the importance of our calculation, since an experimental determination of α — in conjuction with our results — will most certainly serve to tighten the constraints on the possible values of the color-octet matrix elements. In fact, the NRQCD prediction for α is very sensitive to the values of the color-octet matrix elements, and this offers the hope that a measurement of the polarization of J/ψ produced in $b \to J/\psi + X$ will be instrumental in determining $\langle\mathcal{O}_8^\psi(^3S_1)\rangle$, $\langle\mathcal{O}_8^\psi(^1S_0)\rangle$, and $\langle\mathcal{O}_8^\psi(^3P_0)\rangle$.

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Figure Captions

Figure 1. Feynman diagram for the four-quark Fermi interactions $b \to c\bar{c}s$ and $b \to c\bar{c}d$. This interaction describes the short-distance physics for the process $b \to J/\psi + X$. P is the total four-momentum of the $c\bar{c}$ system, and s, d and b are the four-momenta of the corresponding quarks. The $c\bar{c}$ pair is produced initially with quantum numbers $a, {}^{2S+1}L_J$ with $a = 1 \,\mathrm{or}\, 8$. It then hadronizes into a J/ψ particle with helicity λ.

Figure 2. Angular momentum conservation in $b \to c\bar{c}s$ and $b \to c\bar{c}d$. Because the charged-current Fermi-interaction couples only the left-handed parts of the fermion fields, the s and d quarks (being massless) are emitted with negative helicity. Due to angular momentum conservation, therefore, the outcome $\lambda_{c\bar{c}} = +1$ is not allowed. As a result, the $\langle \mathcal{O}_1^{\psi}({}^3S_1) \rangle$ and $\langle \mathcal{O}_8^{\psi}({}^3S_1) \rangle$ contributions to the J/ψ helicity rate are zero for $\lambda = +1$, at lowest order in v^2.

Figure 1

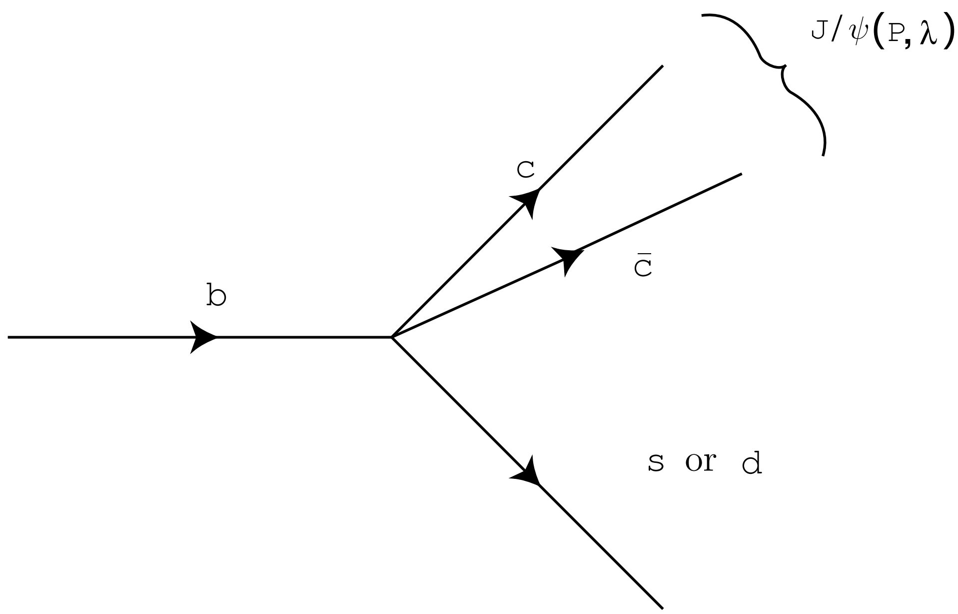

Figure 2